# Individual analysis Assignment

## ME 486C – spring 2019– Smart Helmet
## Buckingham Theorem and Impulse Equation Analysis

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Many football players were injured seriously while they were playing football. The main idea of this assignment is to prevent injuries by applying Buckingham theorem, impulse equation, and the collision process. The main objective of this individual assignment is to maintain safety for the football players. One of the most important client requirement is to maintain safety for the players. Buckingham theorem, the impulse equation, and the collision process analysis are important for the safety purposes in the football field.

Buckingham theorem is one of the most important analysis in the engineering field. Many important dimensionless group were found by using Buckingham theorem. For example, Reynold number is one of the dimension group was found by using the Buckingham theorem. The main purpose of the Buckingham theorem is to make engineering analysis for a specific situation. In another word, this important engineering method helps in studying the engineering physical model. In more specific, this theorem is a very creative theorem where we can create a new equation from the physical model only. In addition, this theorem helps in deeply understand the engineering variables related to the physical mode. The below process is using Buckingham theorem to smart helmet project.

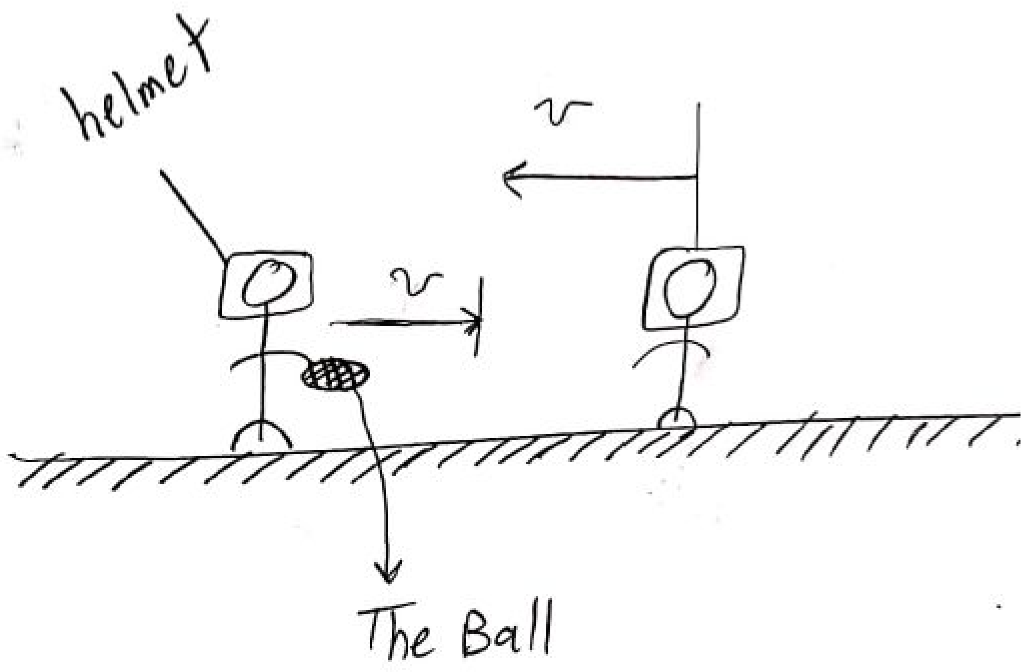


**Figure 1**: describe the physical model of the players

Now we apply the related property to the physical model where:

$$F = f(g, t, v, m)$$

$F = froce\ of\ the\ players$

$t = how\ much\ time\ passes\ ?$

$v = velocity\ of\ the\ players$

$m = \ mass\ of\ the\ players$

$g = acceleration\ due\ to\ gravity$

Let number of dimensionless group $= k$ [3]

Then,

$k = n - j = 5 - 3 = 2$

$n = number\ of\ the\ related\ property$

$j = number\ of\ fundamental\ units$

That means we only need two dimensionless group. Then, we apply the basic principles of Buckingham theorem where:

$\pi_1 = F g^a t^b m^c = [kg]^0 [m]^0 [s]^0$

$Newton\ (N) = \frac{kgm}{s^2}$

Now we can bread down the pi-1 equation as:

$$\pi_1 = \left[\frac{kgm}{s^2}\right] \left[\frac{m^a}{s^{2a}}\right] \left[s^b\right] \left[kg^c\right] = [kg]^0 [m]^0 [s]^0$$

$$= [kg]^{1+c} [m]^{1+a} \left[s^{b-2-2a}\right] = [kg]^0 [m]^0 [s]^0$$

$1 + c = 0 \Rightarrow c =- 1$

$1 + a = 0 \Rightarrow a =- 1$

$b - 2 - 2a = 0 \Rightarrow b = 0$

$$\pi_1 = F g^{-1} t^0 m^{-1} = \frac{F}{gm} = \left[\frac{kgm}{s^2}\right] \left[\frac{s^2}{m}\right] [1] \left[\frac{1}{kg}\right] = first\ dimesnsionless\ group$$

$F = \pi_1 gm,\ \ where\ this\ equation\ represent\ the\ Newton\ second\ law$

$\pi_2 = v g^a F^b t^c = [kg]^0 [m]^0 [s]^0$

$$= \left[\frac{m}{s}\right] \left[\frac{m^a}{s^{2a}}\right] \left[\frac{kg^b m^b}{s^{2b}}\right] \left[s^c\right] = [kg]^0 [m]^0 [s]^0$$

$$= [m]^{1+a+b} [kg]^b [s]^{-1-2a-2b+c} = [kg]^0 [m]^0 [s]^0$$

$1 + a + b = 0 \qquad a =- 1$

$b = 0$

$- 1 - 2a - 2b + c = 0 \qquad c =- 1$

$\pi_2 = vg^{-1}F^0t^{-1} = \frac{v}{gt} = [\frac{m}{s}] * \left[\frac{s^2}{m}\right] * [\frac{1}{s}] = Second\ dimensionless\ group$

$v = \pi_2 gt, \quad where\ this\ represent\ the\ velocity\ function\ of\ the\ physical\ model$

Now we can simply take the derivative to find the acceleration function where:

$a = \frac{dv}{dt} = \pi_2 g, \quad where\ this\ represent\ the\ acceleration\ function$

Now we can plot the curves to recognize the relationship between each of the dimensionless group equation where:

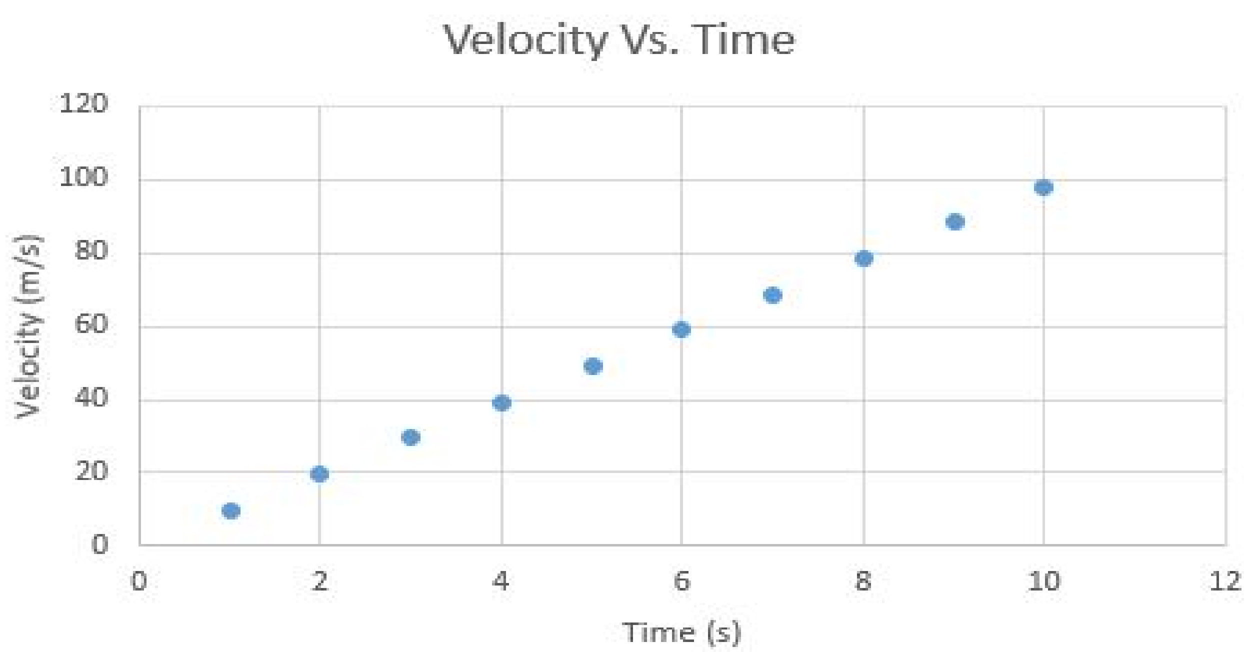


**Figure 2**: the relationship between the velocity and time in this physical model

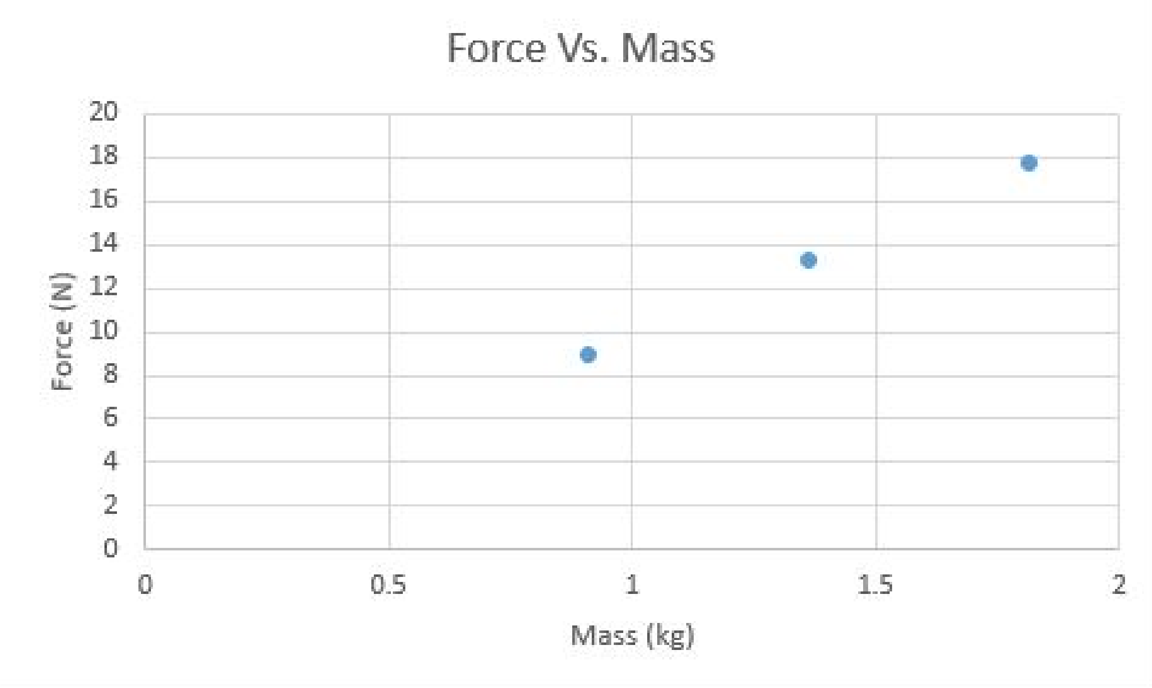


**Figure 3**: The relationship between force and the mass in this physical model

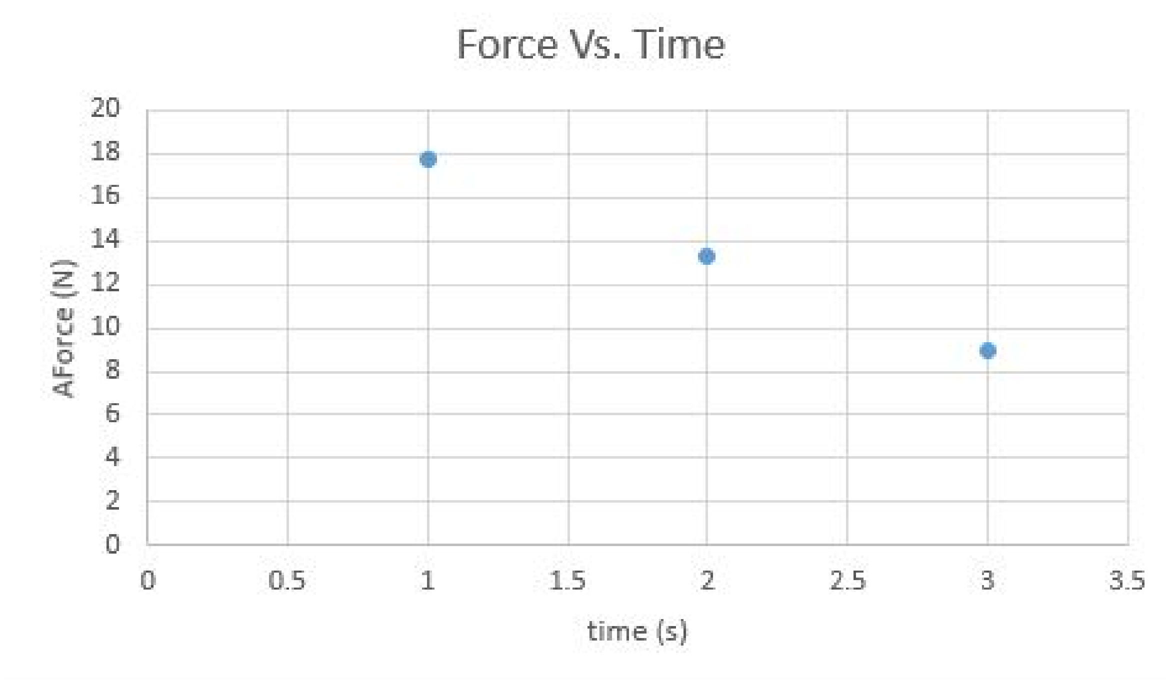


**Figure 4:** The relationship between the force and the time in this physical model

From figure 2 and figure 3, the team understood the relationship between these important related engineering properties. For example, the relationship between the force and time is linear where when the team increase the force, then the mass should be increase as well. Similarly, the relationship between the velocity and time is linearly independent.

In figure 4, the team can recognize the relationship between the force and time in the smart helmet project. The team can conclude from figure 4 that when we increase the time, then the force will be decrease for the collusion process in the football field. The team can figure out the impulse from figure 4 where the area under the curve represent the impulse. The mass values were assumed by the team because the team is still testing, and the team is still adding components to the helmet. However, these mass values are the most common used values in the football field [2].

Now the team can apply the resulted values to the impulse equation where:

$$F_{ave.} * \Delta t = m\Delta v$$

$$F_{ave.} * \Delta t = Impulse = (13.35N) * (1s) = 13.35\ N.s$$

$$m\Delta v = momentum = (1.36kg)\left(9.81\tfrac{m}{s}\right) = 13.34\tfrac{kgm}{s}$$

The team can predict the expected impulse and momentum values in the smart helmet project. If the team applies these values, then the team can increase the safety of the project by trying to shoot for these ranges in the smart helmet project. In addition, the impulse value and the momentum value can help in saving the transmit data in the football field where the team can decrease the damage resulted from the force over the past time in the smart helmet project. Thus, the safety of the players will be increase by knowing the relationship between the properties in this project.

Now we can figure out the head on collision by applying the conservation of momentum equation where:

$$m_1v_1 = m_1v_1' + m_2v_2'$$

$$(1.81kg)\left(9.81\tfrac{m}{s}\right) = (1.81kg)\,v_1' + (1.36kg)v_2'$$

$$v_1' = \tfrac{(m_1-m_2)}{(m_1+m_2)}v_1 = \tfrac{(0.45)}{(3.17)}\left(9.81\tfrac{m}{s}\right) = 1.39\ \tfrac{m}{s}$$

$$(1.81kg)\left(9.81\tfrac{m}{s}\right) = (1.81kg)\,(1.39\tfrac{m}{s}) + (1.36kg)v_2'$$

$$v_2' = 11.21\ \tfrac{m}{s}$$

Now the team is able to understand the collision process in the smart helmet design. These resulted values help the team to understand the collision process before and after the separation of both player’s helmet. The Moreover, this analysis can help in understanding the mechanism of the elastic collision of the smart helmet design. Really, the head on collision help the team in understanding the collision process in smart helmet project.

Finally, Buckingham theorem and the impulse equation are important engineering analysis for the smart helmet project. Buckingham theorem help in understanding the engineering properties associated with the physical model. The impulse equation help in increase the safety for the players by finding the resulted impulse and momentum values. The time should be increase and the force should be decrease over the time, then the team can increase the safety of the smart helmet project. The head collision help the team in understanding the collision process of the players before and after the separation of the both player's helmets.

Work Cited

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